

Michael Meyer

### A day – almost – of math pure

In March 1991 pupils of the 12th grade from Southern Hesse and Northern Baden-Württemberg were first invited to spend a Saturday free of school together, in a friendly atmosphere, to do math und exchange ideas. This initiative was well received from the start. Participants grew in number year after year until they reached record level this year, when three hundred (300) enlisted. The initiative follows the example of the Society for the Advancement of Mathematically Gifted Young People which holds its own Days of Mathematics at the university towns of Karlsruhe, Constance, Ulm and Tübingen.

### A day to learn from

The "Bensheim Day of Mathematics" is more than just one day of competition und lectures, but has grown to be an institution constantly at work in initiating and organizing advancement programmes. All programmes, including those for teachers, are recommended and even supported by the Ministries of Culture in Hesse und Baden-Württemberg. In the region of Bensheim the Day of Mathematics has become well-known as a reliable institution whose additional, intellectually challenging contribution to school mathematics is widely appreciated. Even the general public has increasingly taken notice of the Bensheim Day of Mathematics as is proved by its extensive coverage in the local press.

### A day well organized

In the course of the past four years organization has consolidated. There is a supra-regional committee for mathematical problems which sets the assignments for all similar competitions held at various locations, as well as a local committee responsible for the evaluation of the test papers handed in.

In Bensheim four curators are responsible for the curriculum of advancement programmes, while an industry-sponsored fund guarantees financial security.

The endeavours of the Day of Mathematics are accompanied and reinforced by intensive courses held annually at the universities of Constance or Ulm. In addition to that, there was a modelling week last year at the Centre for Practical Mathematics at the Pfalzakademie Lambrecht.

### A day of challenge

Once a year, the Bensheim Day of Mathematics invites pupils to participate in a competition which is designed to arouse interest in mathematics. All competitions aim at this goal. Our competition, more so than others of

its kind, aims at motivating the broad spectrum of mathematically gifted pupils, rather than concentrating on the highly-gifted few. Moreover, the challenge is mainly team-oriented, and limited to one day.

Pupils who attend a secondary school of the college-preparatory kind, currently in the 12th grade, are eligible. Schools within a distance of approximately 80 kilometers from Bensheim are notified. There, pupils form groups of three to five members whose names may be sent in by their math teachers. The programme of Bensheim Day of Mathematics 1994 may serve as an example:

9:00 checking in

9:30 welcoming address and opening session

9:45 group competition

11:00 individual competition

12:00 lunch and recreation

13:30 high-speed competition

14:30 coffee and cake

15:15 "Exploring the Infinite", lecture by Prof. Dr. Harro Heuser, Karlsruhe

16:30 victory ceremony performed by a representative of the Hesse Ministry of Culture

While the pupils were occupied with the assignments of the competition their teachers joined in a didactic colloquy with a lecture about "Solving problems – but how?" by Dr. Regina Bruder.

All pupils receive diplomas while the best are awarded book prizes in addition. Moreover, twenty pupils are elected for further advancement programmes. As a rule, they get a grant for the modelling week at the Centre for Practical Mathematics which comprises a seven days' stay at the Pfalzakademie Lambrecht, free of charge including expert counselling and guidance.

### A day's challenges – what are they?

As an example of the kind of problems the pupils are confronted with, here are those presented on the Bensheim Day of Mathematics 1994. The following are problems set for the group competition:

**G 1.** (15 points)

From a cube with the edge a eight pyramids are cut off by means of planes that intersect the edges in their midpoint. Determine the volume und the total area of the remaining solid.

**G 2.** (15 points)

The equation

$$||x| + ||y| - 3| - 3| = 1$$

determines in a coordinate diagramm a set of points.

2.1 Why are the horizontal and vertical axis lines of symmetry for the set of points?

2.2 Draw the set of points (unit 1 cm).

**G 3.** (15 points)

For a certain  $t > 0$  the graphs  $F$  and  $G$  of the two functions  $f_t$  and  $g_t$  with

$$f_t(x) = \frac{1}{t} \cdot (t^2 - x^2), x \in \mathbb{R} \text{ und } g_t(x) = x^3 - t \cdot x^2, x \in \mathbb{R}$$

limit exactly a finite region  $A$ . Compute the area of  $A$ .

The following are problems set for the individual competition:

**E 1.** (15 points)

Let  $f_1$  be a function with

$$f(x) = \frac{4}{x^2 + 1}, x \in \mathbb{R}.$$

and let  $K$  be the graph of  $f_1$ .

1.1

By reflection of  $K$  through line  $y = 4$  one gets the graph  $C$  of a function  $g$ . Determine the equation of  $g$ .

1.2

Let be  $u > 0$ . The lines  $x = u$  and  $x = -u$  intersect  $K$  and  $C$  in points  $P, Q, R$  and  $S$ . Determine  $u$  in such a way that the points  $P, Q, R$  and  $S$  form a square.

**E 2.** (10 points)

On three different days the extra-curricular work groups in chess, mathematics and informatics meet. There are 35 participants altogether. They are divided in the following way:

- (a) Exactly 16 persons take part in only one group.
- (b) The chess group consists of 17 participants.
- (c) Exactly eight persons take part in both the math group and the informatics group.
- (d) Exactly 3 persons take part in all three work groups.

How many persons are active solely in the chess group?

**E 3.** (15 points)

Let  $ABCD$  be a quadrilateral with the sides  $|CD| = c$  and the angles  $\sphericalangle BAD = \alpha$ ,  $\sphericalangle CBA = \beta$ ,  $\sphericalangle ADB = 90^\circ$  und  $\sphericalangle ACB = 90^\circ$ . Determine the length of the side  $AB$  in dependence of  $c, \alpha, \beta$ .

**E 4.** (7 points)

What is the last digit of the number  $7^{77}$ ?

The following are problems from the high-speed competition which is done in groups and whose main purpose is simply fun:

**S 1.** (4 points)

Given two concentric circles with radius  $r < R$ . The chord  $AB$  of the larger circle is tangent to the smaller circle. Let  $|AB| = 8$ . How large is the area  $F$  of the circular ring?

**S 2.** (4 points)

The product  
 $STETS \cdot 99999$   
 ends in 705. Decode the number  $STETS$ .

**S 3.** (3 points)

A champagne glass has the shape of a perpendicular circular cone with the basic radius  $r$  and the height  $h$ . Where must the glass be marked to ensure that the champagne, filling the glass to this mark, fills half its volume?

**S 4.** (3 points)

One digit is to be added to the number 10, both left and right, in such a way that the number received is divisible by 72.